Few-Body Syst (2011) 50: 363–365
DOI 10.1007/s00601-010-0190-9

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Issues in the GPD Formulation of DVCS

Received: 29 October 2010 / Accepted: 6 December 2010 / Published online: 24 December 2010

Abstract The kinematics used in computing deeply virtual Compton scattering makes a dramatic difference in terms of the widely used reduced operators that define generalized parton distributions. We analyze this difference at tree-level.

1 Introduction

The amplitudes of deeply virtual Compton scattering (DVCS) can be expressed in terms of generalized parton distributions (GPDs), which complement the knowledge encoded in parton distribution functions [1–3]. Factorization is essential for the treatment of deep inelastic scattering (DIS) and DVCS. It means writing the full scattering amplitude as a convolution of a hard-scattering amplitude to be calculated in perturbation theory, and a soft part embodying the hadronic structure. The use of a hard photon that is far off-shell, say $-q^2 = Q^2 \gg$ any relevant soft mass scale, enables factorization theorems [4] with the identification of the hard scattering amplitude. This paper is devoted to the issue of kinematics in computing the DVCS amplitude in terms of the widely used reduced operators that define GPDs. We do so in the simplest possible setting, namely DVCS on a structure-less spin-1/2 particle.

2 Calculation

We calculate the complete, full DVCS amplitude for the scattering of a massless lepton ℓ off a point-like fermion f of mass m. In the final state, we find the scattered lepton ℓ', the fermion f' with momentum k' and a (real) photon γ', viz $\ell \to \ell' + \gamma^*,\ \gamma^* + f \to \gamma' + f'$. ('complete' means that the amplitude includes the leptonic part and 'full' means that no approximations are made in the calculation of the hadronic amplitude.) The complete amplitude at tree level can be written as

$$\mathcal{M} = \sum_h \mathcal{L}(\{\lambda', \lambda\}h) \frac{1}{q^2} \mathcal{H}(\{s', s\}\{h', h\}). \tag{1}$$

Presented at the 21st European Conference on Few-Body Problems in Physics, Salamanca, Spain, 10 August–3 September 2010.

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The quantities $\lambda'(\lambda)$, $h'(h)$, $s'(s)$ are the helicities of the outgoing(incoming) leptons and photons, and the rescattered(target) fermions, respectively. We write

$$\mathcal{L}(\{\lambda', \lambda\}h) = \bar{u}(\ell'; \lambda')\not{\epsilon}^*(q; h)u(\ell; \lambda), \ \mathcal{H}(\{s', s\}\{h', h\}) = \bar{u}(k'; s')(\mathcal{O}_s + \mathcal{O}_u)u(k; s) \tag{2}$$

where the s- and u-channel operators are the standard text book ones.

We take three kinematics for the momenta of the particles in the hadronic amplitude (q^μ, k^μ incoming, q'^μ, k'^μ outgoing):

δ-Kinematics ($q^+ \to 0$ as $\delta \to 0$)

$$\begin{aligned} q^\mu &= \left(\delta p^+, Q, 0, \frac{Q^2}{2(\zeta+\delta)p^+} + \frac{\zeta m^2}{2x(x-\zeta)p^+}\right), q'^\mu = \left((\zeta+\delta)p^+, Q, 0, \frac{Q^2}{2(\zeta+\delta)p^+}\right), \\ k^\mu &= \left(xp^+, 0, 0, \frac{m^2}{2xp^+}\right), \quad k'^\mu = \left((x-\zeta)p^+, 0, 0, \frac{m^2}{2(x-\zeta)p^+}\right). \end{aligned} \tag{3}$$

$q'^+ = 0$ Kinematics (effectively, '$1+1$' dim.)

$$q^\mu = \left(-\zeta p^+, 0, 0, \frac{Q^2}{2\zeta p^+}\right), \quad q'^\mu = \left(0, 0, 0, \frac{Q^2}{2\zeta p^+} - \frac{\zeta m^2}{2x(x-\zeta)p^+}\right). \tag{4}$$

The momenta k^μ and k'^μ are the same as in case (1).

Nonvanishing q^+ and q'^+ Kinematics (with $m = 0$)

$$q^\mu = \left(-\frac{\zeta}{2}p^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta p^+}\right), \quad q'^\mu = \left(\frac{\zeta}{2}p^+, \frac{Q}{\sqrt{2}}, 0, \frac{Q^2}{2\zeta p^+}\right). \tag{5}$$

The momenta k^μ and k'^μ are the same as in case (1) if the limit $m \to 0$ is taken.

These kinematics correspond to the hard-scattering part of a DVCS amplitude where the fermions are the quarks and p^+ is the plus-component of the momentum of the parent hadron target. In the $\delta \to 0$ limit, the δ-kinematics coincides with the well-known $q^+ = 0$ frame [5] frequently cited in the discussion of the GPD formalism. As taking $q^+ = 0$ will lead to singular polarization vectors in the LF gauge $A^+ = 0$ (see e.g. [6]), we set q^+ to δp^+ and expand all amplitudes in powers of δ, taking the limit $\delta \to 0$ at the very end of the calculation of the complete physical amplitude. The results from these three kinematics are fully in agreement with each other, because in the limit $Q^2 \to \infty$ the invariant amplitudes correspond to the same invariants, s, t, u.

Table 1 Complete amplitudes in δ-kinematics for $\lambda' = \lambda = \frac{1}{2}, s' = s = \frac{1}{2}$

$\{h', h\}$	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\text{Full}}$	$\mathcal{L}\frac{1}{q^2}\mathcal{H}_{\text{Red}}$
$\{1, 1\}$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{4\zeta^2}{\delta^2} + \frac{6\zeta}{\delta} + \frac{3}{2} - \frac{\delta}{4\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x-\zeta}{x}}\left(\frac{2\zeta}{\delta} + 1 - \frac{\delta}{4\zeta}\right)$
$\{1, 0\}$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{-8\zeta^2}{\delta^2} - \frac{4\zeta}{\delta} + 1 - \frac{\delta}{2\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{2\zeta}{\delta} - 1 + \frac{\delta}{4\zeta}\right)$
$\{1, -1\}$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{4\zeta^2}{\delta^2} - \frac{2\zeta}{\delta} + \frac{3}{2} - \frac{5\delta}{4\zeta}\right)$	0
$\sum_h$	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(4 - \frac{2\delta}{\zeta}\right)$	0
$\{-1, 1\}$	$\frac{1}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{4\zeta^2}{\delta^2} - \frac{2\zeta}{\delta} + \frac{1}{2} - \frac{\delta}{4\zeta}\right)$	0
$\{-1, 0\}$	$\frac{1}{Q}\sqrt{\frac{x-\zeta}{x}}\left(\frac{8\zeta^2}{\delta^2} + \frac{4\zeta}{\delta} - 1 + \frac{\delta}{2\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x}{x-\zeta}}\left(\frac{2\zeta}{\delta} + 1 - \frac{\delta}{4\zeta}\right)$
$\{-1, -1\}$	$\frac{1}{Q}\sqrt{\frac{x-\zeta}{x}}\left(-\frac{4\zeta^2}{\delta^2} - \frac{2\zeta}{\delta} + \frac{1}{2} - \frac{\delta}{4\zeta}\right)$	$\frac{2}{Q}\sqrt{\frac{x}{x-\zeta}}\left(-\frac{2\zeta}{\delta} + 1 - \frac{3\delta}{4\zeta}\right)$
$\sum_h$	0	$\frac{1}{Q}\sqrt{\frac{x}{x-\zeta}}\left(4 - \frac{2\delta}{\zeta}\right)$

The reduced hadronic operators used in the formulation of GPDs are defined as the limits $Q \to \infty$ of the operators $\mathcal{O}_s$ and $\mathcal{O}_u$, Eq. (2), and found to be,

$$\mathcal{O}_s|_{\text{Red}} = \frac{\not\epsilon^*(q';h')\gamma^+\not\epsilon(q;h)}{2p^+}\frac{1}{x-\zeta}, \quad \mathcal{O}_u|_{\text{Red}} = \frac{\not\epsilon(q;h)\gamma^+\not\epsilon^*(q';h')}{2p^+}\frac{1}{x}. \tag{6}$$

These reduced propagators contain the nilpotent Dirac matrix γ^+ only, which kills the singular parts of the polarization vectors, namely $\epsilon^-(q;h)\gamma^+$. This is the reason for disregarding the singularities in the polarization vectors in $q^+ = 0$ kinematics. However, the leptonic part $\mathcal{L}$ of the complete amplitude is also singular. Consequently, the complete amplitude calculated with the reduced hadronic part and taking into account the transverse polarizations only, is wrong, even in the limit $Q \to \infty$.

Table 1 clearly shows that the reduced amplitudes and the full ones disagree. We have checked that the same disagreement occurs in the nonvanishing q^+ and q'^+ kinematics given by Eq. (5), although for the kinematics without any transverse component, e.g. Eq. (4), the reduced amplitudes and the full ones do agree. Upon convoluting the leptonic and hadronic amplitudes to obtain the complete ones, the singular $1/\delta$-terms cancel in δ-kinematics, but the full and reduced hadronic amplitudes do not produce the same complete ones. Moreover, if the contribution of the longitudinal polarization of the virtual photon is neglected, the singular parts do not cancel out either. So, the contribution of the longitudinal part is not suppressed by a factor $1/Q$ compared to the contributions of the transversely polarized photons. Therefore, it must not be neglected in the kinematics given by Eqs. (3) and (5), where the photons carry transverse momenta of order Q.

We conclude that in any kinematics where the transverse components of the momenta are of order Q the full hadronic amplitudes and the reduced ones do not agree, even in the limit $Q \to \infty$, which means that the calculations of the DVCS amplitudes using the GPD cannot be trusted in this kinematics. In addition, the contribution of the longitudinally polarized virtual photon is not down by one order in Q but even plays the role of cancelling the singular parts.

This work is supported in part by the U.S. Department of Energy (No.DE-FG02-03ER41260).

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